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# STATISTICAL ANALYSIS OF PHOTOGRAPHIC METEOR DATA, PART II: VERNIANI'S LUMINOUS EFFICIENCY AND SUPPLEMENTED WHIPPLE WEIGHTING

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#### ABSTRACT

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This report is Part II in a series of four reports on the statistical analysis of the Hawkins and Southworth random sample of 285 sporadic photographic meteors. The four parts of the analysis comprise the combinations from two alternative formulations for meteoroid mass and two alternative formulations for data weighting. Parts I and II have the same weighting as a function of air-entry velocity and other parameters. But the surprisingly large disparity between the results with Öpik's luminous efficiency suggests the possibility of considerable bias from weighting inversely with the 1.5-power of velocity.

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#### ACKNOWLEDGEMENTS

The computations supporting this MSFC in-house effort were programmed by Mrs. Sylvia Bryant for a GE 225 digital computer. Encouragements for the extension of this analysis to include alternative combinations of physical theory and data weighting and other helpful suggestions were given by Dr. Julius Dohnanyi from Bellcomm, Inc.

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## **DEFINITION OF SYMBOLS**

Symbol	Definition
m	Meteoroid mass in grams in space
v	Meteoroid velocity before deceleration in the atmosphere
$^{eta}_{\mathbf{e}}$	Celestial latitude of corrected radiant in degrees
e	Meteoroid heliocentric orbital eccentricity
λ	Elongation of the true radiant from the apex of the earth's way in degrees
$\mathbf{f_t}$	Terrestrial weighting function in consideration of the population of meteoroids that actually encounter the earth
$\mathbf{f_s}$	Spatial weighting function in consideration of the population of meteoroids in space that may or may not encounter the earth
log	Logarithm to base ten
$X_1, \ldots, X_5$	Representation of log m, log v, $ \beta_e $ , e, and $\lambda$ , respectively.
$\overline{X}_i$ , $s_i$	Sample weighted mean, and weighted standard deviation for the variable $X_i$ where $i = 1, \ldots, 5$
r <sub>ij</sub>	Sample weighted correlation coefficient between $X_i$ and $X_j$ for i, $j = 1, \ldots, 5$
R	The determinant of the sample weighted correlation coefficients $r_{ij}$ for i, $j = 1, \ldots, 5$
${f R}_{f ij}$	Cofactor of the element r in the determinant R
s e	Standard error of the estimate of $\mathbf{X_1}$ from the equation of the regression plane

## DEFINITION OF SYMBOLS (Concluded)

Symbol	Definition
r <sub>1•2345</sub>	Multiple correlation coefficient for $X_1$ in relation to $X_2, \ldots, X_5$
<sup>r</sup> ij•klm	Partial correlation coefficient between $X_i$ and $X_j$ with $X_k$ , $X_l$ , and $X_m$ held fixed, where i, j, k, l, m = 1,, 5
×	Data point for log m not less than the weighted median log m
•	Data point for log m less than the weighted median log m
$\mathbf{M}_{\mathbf{p}}$	Meteor absolute photographic magnitude
$^{\mathbf{F}}{}_{\mathbf{M}}{}_{\mathbf{p}}$	Mean number of sporadic and stream meteors per second per square meter of level surface with absolute photographic magnitude equal to or less than ${\rm M}_{\rm p}$
$\mathbf{F}_{>}$	Mean number of sporadic and stream meteoroids per second per square meter of level surface with mass equal to or greater than m grams
Fmv	Mean number of sporadic and stream meteoroids per second per square meter of level surface with momentum equal to or greater than mv gram kilometers per second
$\mathrm{F_{mv^2}}$	Mean number of sporadic and stream meteoroids per second per square meter of level surface with kinetic energy equal to or greater than $mv^2$ gram kilometers <sup>2</sup> second <sup>-2</sup>
$\mathrm{F_{mv^{3/2}}}$	Mean number of sporadic and stream meteoroids per second per square meter of level surface with the geometric mean of momentum and kinetic energy equal to or greater than $\mathrm{mv}^{3/2}$ gram kilometers $^{3/2}$ second $^{-3/2}$
Z	Zenith angle to meteor radiant in radians

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# STATISTICAL ANALYSIS OF PHOTOGRAPHIC METEOR DATA, PART II: VERNIANI'S LUMINOUS EFFICIENCY AND SUPPLEMENTED WHIPPLE WEIGHTING

#### SUMMARY

This report is Part II in a series of four reports on the statistical analysis of the Hawkins and Southworth random sample of 285 sporadic photographic meteors. The four parts of the analysis comprise the combinations from two alternative formulations for meteoroid mass and two alternative formulations for data weighting. Parts I and II have the same weighting as a function of air-entry velocity and other parameters. But the surprisingly large disparity between the results with Öpik's luminous efficiency suggests the possibility of considerable bias from weighting inversely with the 1.5-power of velocity.

#### INTRODUCTION

#### Justification and Purpose

The technology of meteoroids and of their interaction with fields and with natural and artificial bodies in space is of considerable scientific and engineering interest. Each of the several sources of data continues to be of difficult interpretation because of indirection, extrapolation, and bias in random samples caused by physical selectivity. The difficulty of using a sample of photographic meteor data to infer the flux distributions of meteoroids as functions of dynamic parameters is that the results are sensitive both to the luminous efficiency formulation as a function of velocity and to the manner of weighting of the data. A study of the statistical consequences of such different alternatives should alleviate the decision problem. This report, with Part I [1] of this series, will present in more detail some results that were mentioned in two recent papers [2, 3].

#### Method and Notation

A multivariable statistical analysis is made with weighting functions of meteor height, velocity, celestial latitude, zenith angle, and earth-encounter probability. The sample is then equally divided with respect to an intermediate mass value, and weighted cumulative distributions for several parameters are plotted for the two gradations with respect to mass, jointly or separately. The analysis is repeated without the weighting with respect to the earth-encounter probability. All weighting factors are adjusted so that the sum for the sample is equal to the sample size. All flux values are cumulative with respect to the indicated parameter (e.g., mass, momentum, etc.) and are in numbers per second per square meter of level surface. All logarithms are for base ten. Statistical notation is according to Hoel [4].

#### Scope

The same data sample and the same weighting as in Part I [1] of this series are used in the present analysis; i.e., Hawkins and Southworth's [5, 6,] random sample of 285 sporadic photographic meteors described in Part I [1]. But instead of calculating the meteor mass values as in Part I [1] by using Öpik's [7] physical theory of meteors, Hawkins and Southworth's [6] tabulated values are used. These mass values were computed by Hawkins and Southworth [6] using Hawkins [8] "short trail method," which is said to presuppose Verniani's [9] meteor luminous efficiency directly proportional to velocity.

The weighting function for the present analysis was developed in Part I [1]. After weighting inversely with the square of meteor height at maximum brilliance, inversely with the 3/2 power of the air-entry velocity, inversely with the apparent fraction of the circle of celestial latitude through the meteor radiant, and inversely with Opik's [10] earth-encounter probability, it was found that a weighting also inversely with exp (0.18Z) maximized symmetry with respect to the ecliptic plane (where Z is the zenith of the meteor radiant in radians). Because the meteors with radiants more than 42° below the ecliptic were obscured by the horizon, the meteors with radiants more than 42° above the ecliptic were given double weight for arithmetic considerations of celestial latitude. This "spatial" weighting factor  $\boldsymbol{f}_{\mathrm{S}}$  is used in consideration of a population of meteoroids in space regardless of whether or not they may encounter the earth. The "terrestrial" weighting factor  $\mathbf{f}_t$  is similar to  $\mathbf{f}_s$  except that  $\mathbf{f}_t$  does not involve  $\ddot{\mathbf{O}}$ pik's [10] probability that a meteoroid with given orbital parameters will encounter the earth during one revolution of the particle. The meteor data and values of the weighting functions were tabulated in Part I [1].

#### DISCUSSION OF RESULTS

The results with the set of mass values used in the present analysis are quite different from those reported in Part I [1] with the mass values calculated from Opik's [7] physical theory. The results of the multivariable statistical analysis are tabulated in Appendix I. For example, with "terrestrial" weighting,  $r_{12}$  (direct correlation between the logarithms of mass and velocity) changed from 0.01 to -0.69, while the corresponding partial correlation  $r_{12 cdots 345}$  changed from 0.10 to 0.41;  $r_{14}$  (direct correlation between log mass and eccentricity) changed from 0.11 to -0.45, while the corresponding partial correlation  $r_{14 cdots 235}$  changed only from -0.14 to -0.23; and  $r_{15}$  (direct correlation between log mass and elongation from the apex of the earth's way) changed from 0.11 to 0.54, while the corresponding partial correlation  $r_{15 cdots 234}$  changed only from -0.02 to -0.07. The indicated changes in  $r_{14}$  and  $r_{15}$  appear to reflect the changes in  $r_{12}$  and  $r_{12 cdots 345}$  through  $r_{24} = 0.81$  and  $r_{25} = -0.64$ .

With "spatial" weighting, the median value  $\log m = -1.88$  divides the sample into two parts with a lower weighted mean  $\log m = -2.35$  and an upper weighted mean  $\log m = -1.38$ . With "terrestrial" weighting the corresponding median, lower mean, and upper mean for  $\log m$  are -1.46, -1.98, and -1.17, respectively. Figures 1 through 5 each show cumulative distributions of parameters over the separate mass-subsets.

Cumulative distributions of eccentricity are shown in Figures 1 and 2 for "spatial" and "terrestrial" weighting, respectively. The separation of the "high" mass cumulative distribution from that for the meteors of "low" mass depends more on the ordinary correlation  $r_{14}$  than on the corresponding partial correlation  $r_{14 \cdot 235}$ ; e.g., Figures 1 and 2 show that meteors of low mass tend to have high eccentricity in agreement with the negative correlations  $r_{14} = -0.47$  and -0.45, respectively, from Appendix I.

Figures 3 and 4 show for the cumulative distribution of arithmetic celestial latitude of meteor radiant the same as Figures 1 and 2, respectively, showed for the cumulative distribution of eccentricity. The barely separable plots in Figures 3 and 4 reflect the numerically low correlations  $\mathbf{r}_{13}=0.10$  and 0.04, respectively, from Appendix I. But the widely separated "terrestrially" weighted cumulative distributions of velocity in Figure 5 reflect the strongly negative correlation  $\mathbf{r}_{12}=-0.69$  from Appendix I.

Figures 6 through 10 show weighted whole-sample log cumulative distributions with respect to the logarithms of dynamic parameters such as mass, momentum, etc. In each case the plots seem to be approximately linear over the large-mass half of the sample weight. Presumably, the smaller masses are not adequately represented, and therefore should be ignored in Figures 6 through 10.

Figure 6 shows that the logarithm of the "spatially" weighted cumulative distribution of log mass has a unit negative slope with respect to log mass for the present set of mass values, just as was found in Part I [1] with the other set. Also the corresponding slope with "terrestrial" weighting in Figure 7 has the same value (-1.34) as was found in Part I [1], with the other set of mass values, and as was found previously by Hawkins and Upton [11] with a somewhat smaller sample and different weighting.

Figures 8 through 10 show that the slope (-1.34) in Figure 7 is invariant with respect to replacing log mass with the logarithms of momentum, kinetic energy, and the geometric mean of momentum and kinetic energy, respectively. The derivation in Appendix II indicates that this is the result that should be found if mass and velocity were statistically independent. But the correlation is not numerically small in the present case; and different slopes were found in Part I [1] with numerically smaller correlation. No explanation for this effect has been found.

Any ordinate in Figures 7 through 10 is converted into log cumulative mean total flux by subtracting -15. 18 (the logarithm of the area-time exposure product in square meter seconds) and adding 0.08 (the logarithm of the factor by which mean total flux exceeds mean sporadic flux); see Part I [1]. The results are shown in Figure 11.

#### CONCLUSIONS

The results of the present analysis differ more widely from those in Part I [1] with the same weighting than had been expected. This is caused by the interacting roles of mass and velocity, and possibly to a bias with respect to velocity that may not be appropriately reduced by the weighting function that is inversely proportional to the 3/2 power of velocity. The next run of this analysis, with the Hawkins and Upton's [11] weighting as a function of magnitude above the limit of the photographic plate, instead of weighting as a function of velocity, is now expected to give more convincing results.

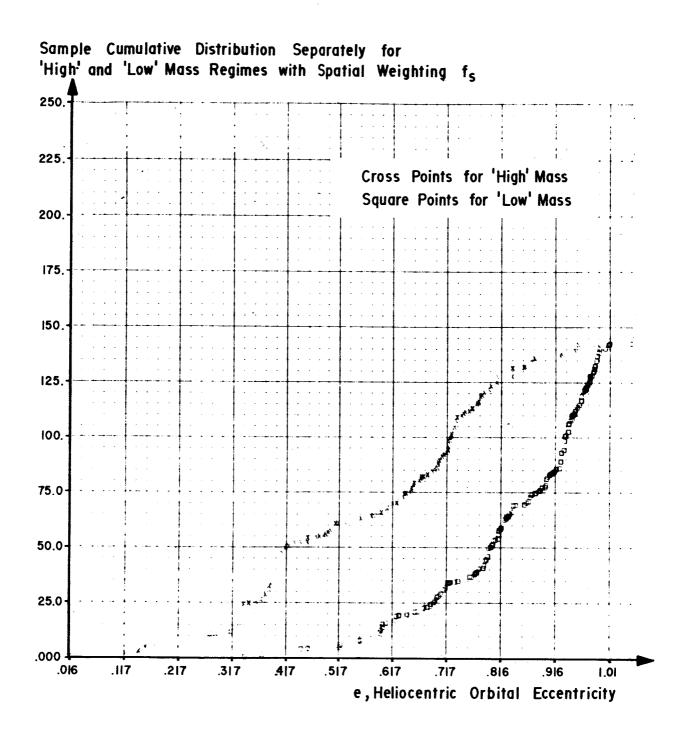


FIGURE 1. SPATIALLY WEIGHTED DISTRIBUTIONS OF ECCENTRICITY

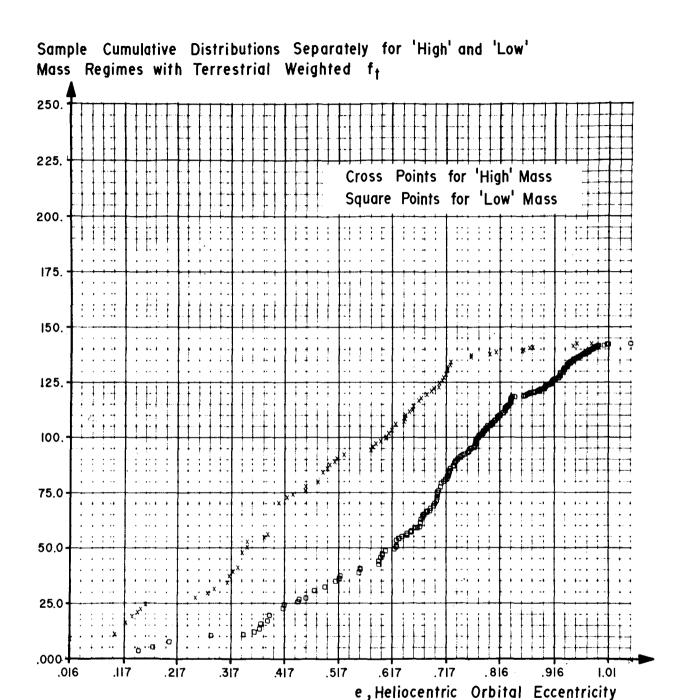


FIGURE 2. TERRESTRIALLY WEIGHTED DISTRIBUTIONS OF ECCENTRICITY

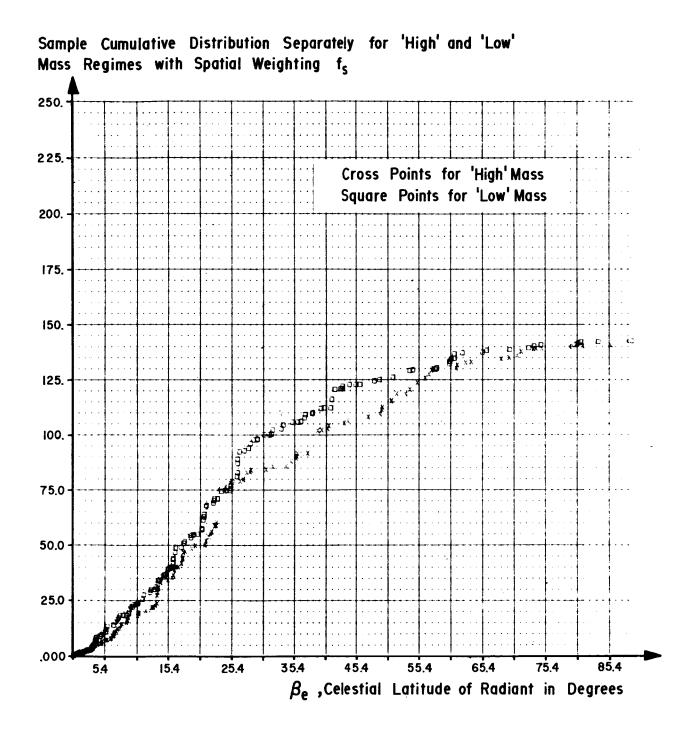


FIGURE 3. SPATIALLY WEIGHTED DISTRIBUTIONS OF CELESTIAL LATITUDE OF RADIANT

Sample Cumulative Distribution Separately for 'High' and 'Low' Mass Regimes with Terrestial Weighting  $f_{\mbox{\scriptsize t}}$ 

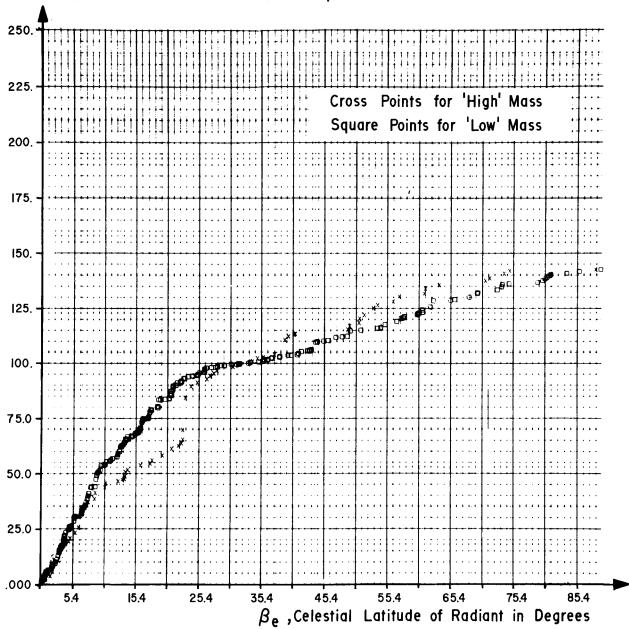


FIGURE 4. TERRESTRIALLY WEIGHTED DISTRIBUTIONS OF CELESTIAL LATITUDE OF RADIANT

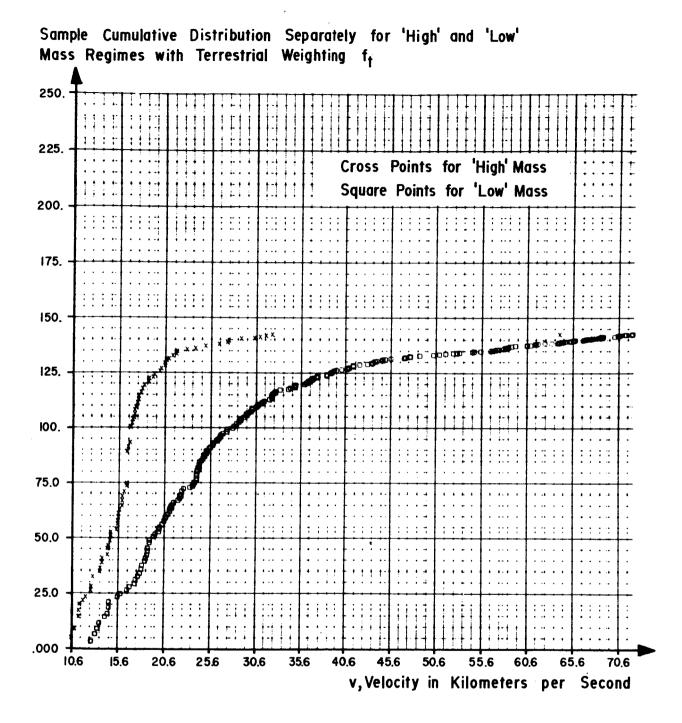
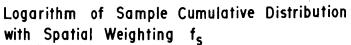


FIGURE 5. TERRESTRIALLY WEIGHTED DISTRIBUTION OF AIR-ENTRY VELOCITY



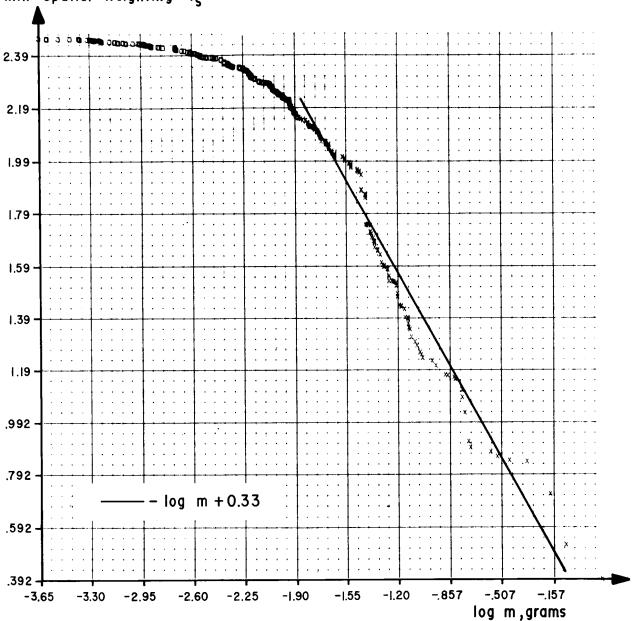


FIGURE 6. SPATIALLY WEIGHTED DISTRIBUTION OF METEOROID MASS

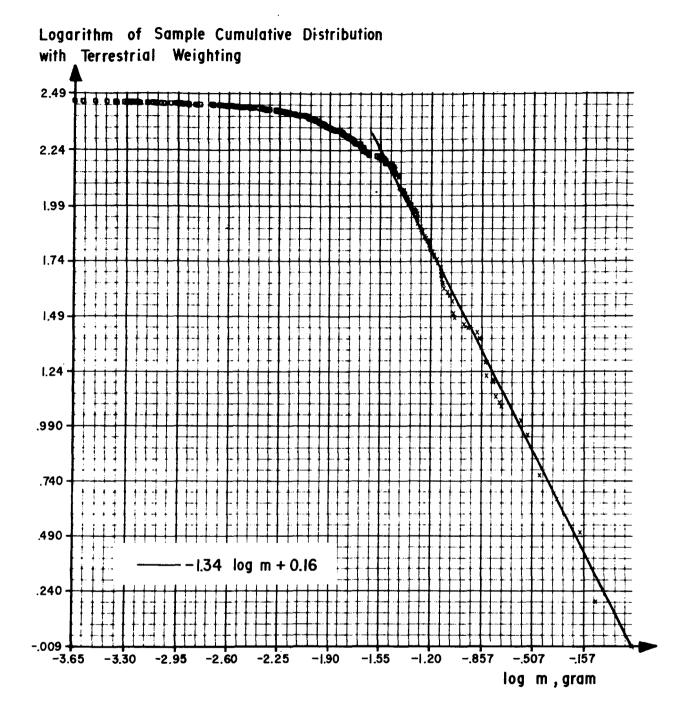


FIGURE 7. TERRESTRIALLY WEIGHTED DISTRIBUTION OF METEOROID MASS

# Logarithm of Sample Cumulative Distribution with Terrestrial Weighting $f_{t}$

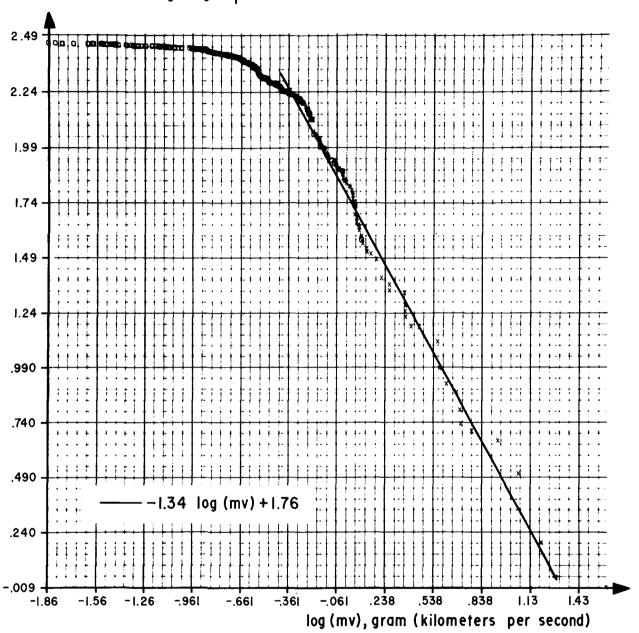


FIGURE 8. TERRESTRIALLY WEIGHTED DISTRIBUTION OF METEOROID AIR-ENTRY MOMENTUM

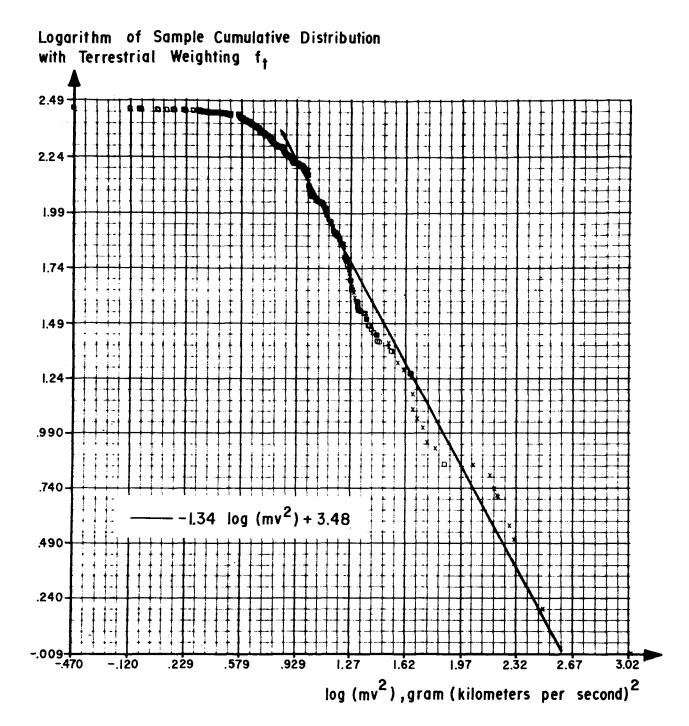


FIGURE 9. TERRESTRIALLY WEIGHTED METEOROID AIR-ENTRY KINETIC ENERGY

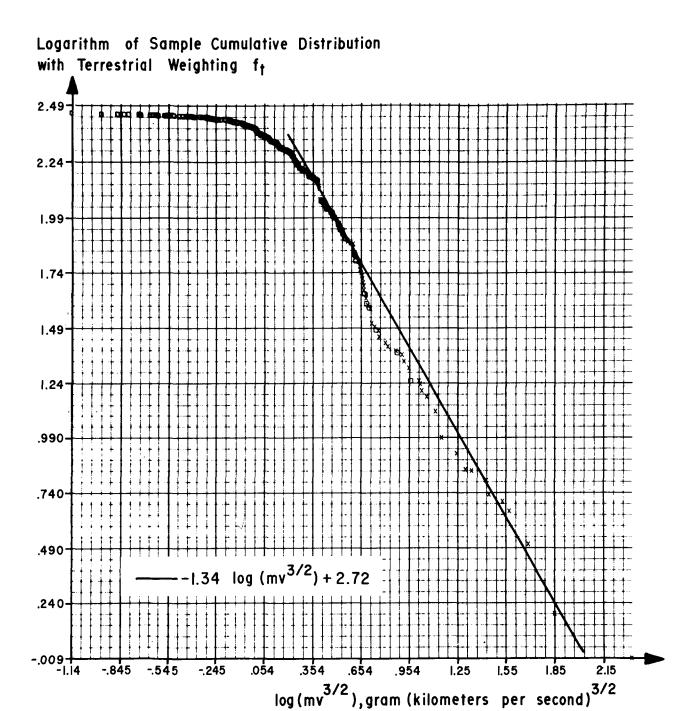


FIGURE 10. TERRESTRIALLY WEIGHTED DISTRIBUTION OF THE GEOMETRIC MEAN OF METEOROID AIR-ENTRY MOMENTUM AND KINETIC ENERGY

$$log F_{Mp} = 0.537 Mp - 13.81$$

$$log F_{>} = -1.34 log m -14.94$$

$$log F_{mv} = -1.34 log (mv) -13.34$$

$$log F_{mv^2} = -1.34 log (mv^2) -11.62$$

$$log F_{mv}^{3/2} = -1.34 (mv^{3/2}) -12.38$$

Terrestrial Weighting f <sub>t</sub>	Spatial Weighting f <sub>s</sub>
0.405 for log m vs. log v	0.135 for log m vs. $ oldsymbol{eta_e} $
0.195 for log m vs. $ oldsymbol{eta_e} $	-0.235 for log m vs. e
-0.435 for log v vs. $ oldsymbol{eta_e} $	0.521 for $ oldsymbol{eta_e} $ vs. e
0.835 for log v vs. $\lambda$	-0.797 for $\lambda$ vs. e

FIGURE 11. SUMMARY RESULTS

# APPENDIX I. NUMERICAL RESULTS FROM THE MULTIVARIABLE STATISTICAL ANALYSIS. $X_1,\ldots,X_5=\log m, \log v,$ $|\beta_e|$ , e, AND $\lambda$ , RESPECTIVELY

STATISTICAL PARAMETER	WITH SPATIAL WEIGHTING f	WITH TERRESTRIAL WEIGHTING f <sub>t</sub>
$\overline{X}_1$	-1.8652	<b>~1.</b> 5691
$\overline{\mathbf{X}}_2$	1. 4240	1, 2870
$\overline{\mathrm{x}}_{\scriptscriptstyle 3}$	28. 9383	25. 8878
$\overline{X}_4$	0.7040	0. 5505
$\overline{\mathrm{X}}_{5}$	83, 1402	97. 1172
$\mathbf{s_i}$	<b>0. 62</b> 85	0 <b>.</b> 5 <b>4</b> 33
$\mathbf{s_2}$	0.1793	0.1684
$\mathbf{s_3}$	18. 9195	22, 2727
s <sub>4</sub>	0.2214	0.2418
$\mathbf{s_5}$	21, 2353	24, 4780
r <sub>11</sub>	1,0000	1.0000
$\mathbf{r}_{12}$	-0.7101	-0.6879
$\mathbf{r_{i3}}$	0.1040	0.0380
r <sub>14</sub>	-0.4714	-0.4529
$\mathbf{r_{i5}}$	0. 6828	0.5429
$\mathbf{r_{22}}$	1,0000	1.0000
${f r_{23}}$	-0. 1242	0.0465
$\mathbf{r_{24}}$	0.8324	0.8108
$\mathbf{r_{25}}$	-0.8258	-0. 6369
$\mathbf{r}_{33}$	1.0000	1,0000

STATISTICAL PARAMETER	WITH SPATIAL WEIGHTING f	WITH TERRESTRIAL WEIGHTING f <sub>t</sub>
r <sub>34</sub>	-0. 2718	-0. 1343
$\mathbf{r_{35}}$	0. 1196	-0. 0606
r <sub>44</sub>	1. 0000	1,0000
r <sub>45</sub>	<b>-0. 449</b> 6	-0. 1649
$\mathbf{r_{55}}$	1.0000	1,0000
R	0.0132	0, 0313
R <sub>11</sub>	0.0300	0.0658
R <sub>12</sub>	-0.0361	-0.0763
R <sub>13</sub>	-0.0032	-0.0100
R <sub>14</sub>	0. 0155	0. 0323
R <sub>15</sub>	0.0027	0. 0070
$ m R_{22}$	0.3690	0. 5399
$ m R_{23}$	0.0392	0.0637
R <sub>24</sub>	-0. 2166	-0. 3726
R <sub>25</sub>	-0.1872	-0. 2371
$ m R_{33}$	0.0185	0. 0396
$\mathbf{R_{34}}$	-0.0270	-0.0483
$ m R_{35}$	-0.0202	-0.0248
R <sub>44</sub>	0. 1459	0. 2976
R <sub>45</sub>	0. 1059	0. 1678

STATISTICAL PARAMETER	WITH SPATIAL WEIGHTING f	WITH TERRESTRIAL WEIGHTING f
${f R}_{55}$	0. 1210	0. 1493
${\rm s}_{\rm e}$	0.4173	0.3748
r <sub>1•2345</sub>	<b>0.</b> 7478	0.7240
r <sub>12•345</sub>	0. 3426	0.4051
r <sub>13•245</sub>	0. 1353	0. 1951
r <sub>14</sub> . <sub>235</sub>	-0. 2346	-0.2308
r <sub>15• 234</sub>	-0.0445	-0.0704
r <sub>23• 145</sub>	<b>-0.</b> 4748	-0.4352
r <sub>24• 135</sub>	0 <b>,</b> ·9332	0, 9296
r <sub>25• 134</sub>	0.8870	0,8350
r <sub>34</sub> . <sub>125</sub>	0 <b>.</b> 5209	0.4451
r <sub>35• 124</sub>	0.4282	0.3219
r <sub>45• 123</sub>	-0.7966	-0.7958

### APPENDIX II. SLOPE OF LOG-CUMULATIVE-FLUX VS LOG-MASS, LOG-MOMENTUM, ETC. FOR METEOROIDS ASSUMING STATISTICAL INDEPENDENCE OF MASS AND VELOCITY

Let F,, the meteoroid cumulative-flux with respect to mass m, be

$$F_{>} = 10^{\beta_6} m^{\beta_2}$$
 , (1)

where  $\beta_6$  and  $\beta_2$  are constants. A meteoroid with mass m not less than some limiting value  $m_L$ , but otherwise random, has the following probability density function for m:

$$f(m) = \left(\frac{dF_{>}}{dm}\right) / \int_{m_{I}}^{\infty} \left(\frac{dF_{>}}{dm}\right) dm = -\beta_2 m_{L}^{-\beta_2} m^{\beta_2 - 1}$$
 (2)

A meteoroid with m > m $_{\rm L}$  that has a particular velocity v, but that is otherwise random, can satisfy the requirement

$$mv^{n} > M \tag{3}$$

with probability Pc,

$$P_{c} = \int_{Mv}^{\infty} f(m) dm = \left(\frac{Mv^{-n}}{m_{L}}\right)^{\beta_{2}}, \qquad (4)$$

where n is a constant; i.e., n=1 for momentum, 2 for kinetic energy, etc. Then, if v is randomly distributed in the interval  $v_L < v < v_u$  and has some continuous probability density function f(v), then the probability P that a meteoroid with  $m > m_L$  but otherwise random will satisfy  $mv^n > M$  is found from

$$kP = \int_{v_L}^{v_u} \left(\frac{Mv^{-n}}{m_L}\right)^{\beta_2} f(v) dv = \left(\frac{M}{m_L}\right)^{\beta_2} \int_{v_L}^{v_u} v^{-n\beta_2} f(v) dv , \qquad (5)$$

where k is constant is determined by the further condition that P=1 when  $M=m_{T_i}v_{T_i}^n$ ; i.e.,

$$k = v_L^{n\beta_2} \int_{v_L}^{v_u} v^{-n\beta_2} f(v) dv . \qquad (6)$$

Then, by Equations (5) and (6),

$$\mathbf{P} = \begin{pmatrix} \mathbf{m}_{\mathbf{L}} \mathbf{v}_{\mathbf{L}} \end{pmatrix} - \beta_{2} \mathbf{M}^{\beta_{2}} \tag{7}$$

Therefore, because M in Equation (7), and m in Equation (1) have the same exponent, the slope of log-cumulative-flux vs log-mass should be invariant with respect to the substitution of momentum or energy for mass when mass and velocity are statistically independent.

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# STATISTICAL ANALYSIS OF PHOTOGRAPHIC METEOR DATA, PART II: VERNIANI'S LUMINOUS EFFICIENCY AND SUPPLEMENTED WHIPPLE WEIGHTING

By Charles C. Dalton

The information in this report has been reviewed for security classification. Review of any information concerning Department of Defense or Atomic Energy Commission programs has been made by the MSFC Security Classification Officer. This report, in its entirety, has been determined to be unclassified.

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